Non-Linear Relationship

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Non-linear relationship is fundamental to most physical and statistical phenomena and their study is important to fully understand the world around.

Linear relationships are the easiest to understand and study and a number of very important physical phenomena are linear. However, it doesn't cover the whole ambit of our mathematical techniques and non-linear relationships are fundamental to a number of most important and intriguing physical and social phenomena around.

Examples of Non-Linear Relationships

As their name suggest, non-linear relationships are not linear [1], which means by doubling one variable [2], the other variable will not double.

There are an endless variety of non-linear relationships that one can encounter. However, most of them can still fit into other categories, like polynomial, logarithmic, etc.

Examples:

- The side of a square and its area are not linear. In fact, this is a quadratic relationship. If you double the side of a square, its area will increase 4 times.
- While charging a capacitor, the amount of charge and time are non-linearly dependent. Thus the capacitor is not twice as charged after 2 seconds as it was after 1 second. This is an exponential relationship.

Studying Non-Linear Relationships

Even though non-linear relationships are much more complicated than linear ones, they can be studied in their own right. If you are studying these, you should first see if they fit any standard shapes like parabolas or exponential curves. These are commonly occurring relationships between variables [3].

For example, the pressure and volume of nitrogen during an isentropic expansion are related as PV1.4 which is highly non-linear but fits neatly into this equation.

Next, a number of non-linear relationships are monotonic in nature. This means they do not oscillate and steadily increase or decrease. This is good to study because they behave qualitatively like linear relationships for a number of cases.

Approximations

A linear relationship is the simplest to understand and therefore can serve as the first approximation of a non-linear relationship. The limits of validity [4] need to be well noted. In fact, a number of phenomena were thought to be linear but later scientists realized that this was only true as an approximation.

Consider special theory of relativity that redefined our perceptions of space and time. It gives the full non-linear relationship between variables. They can very well be approximated to be linear in Newtonian mechanics as a first approximation at lower speeds. If you consider momentum, in Newtonian mechanics it is linearly dependent on velocity. If you double the velocity, the momentum will double. However, at speeds approaching those of light, this becomes a highly non-linear relationship.

Some of the greatest scientific challenges need the study of non-linear relationships. The study of turbulence, which is one of the greatest unsolved problems in science and engineering, needs the study of a non-linear differential equation.

Links
[1] https://m.explorable.com/linear-relationship
[2] https://m.explorable.com/research-variables
[3] https://m.explorable.com/relationship-between-variables
[4] https://m.explorable.com/statistical-validity